Queuing in Whaling※

By

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1. Introduction

A series of anayses of whaling records was undertaken to gain information on the whaling operation viewed in the light of the theory of queues. The operation of whaling involves, in the case of the mothership fleet whaling, hunting whales, towing the game, landing it on the mothership and dissecting the carcasses. From the standpoint of the queuing theory harpoonig a whale is the arrival of a unit and the succeeding part of the operation is the service made up of several stages.

The only record the author used for the operations research was the official whaling record of Japan.* It contains data about the time of the individual catch and its landing, but none about the rest of the operation. The present paper deals with the two items; the occurrence of the catch as the input of a waiting-line system and dissecting the carcass as a service of one stage.

In the following sample A denotes a whaling campaign in the North Pacific and samples B and C the two independent campaigns in the Antarctic Sea. The game was sperm whale with sample A and fin whale with samples B and C.

2. Occurrence of catch

The time between catches was statistically analyzed. Both the mean and the variance were computed. The results are shown in Table 1. The mean occurrence rates are also shown in the same table. As is known from the numerical values in Table 1, the ratios of the variance to the square of the mean time are roughly equal

Table 1. Time between catches.

Sample	Mean τ	Variance	Mean occurrence rate λ
A	0.41	0.22	2.11
B	0.51	0.29	1.88
C	0.42	0.18	2.24

Unit of time : 1 hr.

※ Contribution from the Shimonoseki College of Fisherles, No. 364.
Received Jan. 18, 1962.
* The author wishes to express his appreciation to the Fisheries Agency for making this material available.

to unity. This fact suggests that the distribution of the time between catches is exponential. To examine this suggestion the chi-square test of fitness was applied to the working hypotheses. In the computation of the test the sample parameters were used as the parameters of the hypothetical distributions. The result was favourable to sample C with the risk of 15 % defective, but not to the others. The whaling data used contain catches, big and small, and catches under various weather conditions. Hence the data do not seem to be homogeneous. Taking it into account, the dubious discrepancies of the sample distributions from the hypothetical ones might be insignificant, though the risk of defective amounts to 75 % with samples B and C. The distribution of time between catches may at any rate be called exponential. Let r be the mean occurrence rate. Then the exponential distribution under consideration can be expressed by

$$p(t)=e^{-rt},$$

where p(t) is the probability that the time between catches will be t. It is noted in passing that r is equal to the reciprocal of mean time interval τ. If the distribution of time between occurrences is exponential, it follows that the probability c(n) of exactly n catches in the unit time is Poissonian distributed with the parameter λ defined by the expression

$$c(n)=\frac{\lambda^n}{n!}e^{-\lambda}.$$

From the property of Poisson distribution λ is the mean number of catches in the unit time, so that λ is $1/\tau$. In Table 2 λ's obtained from the data are shown together with respective $1/\tau$.

Table 2. Mean rate of catch.

Sample	Average catch per hour λ	$1/\tau$	Sample size
A	2.11	2.45	74
B	1.88	1.96	234
C	2.24	2.38	221

3. Service time

The service rendered to the game consists of several stages. But in this article the service to be studied is limited to a single stage, that is, dissecting the carcass. The time of the service preceding it is included in the waiting time and the service following dissection is left out of consideration.

The service time for the individual whale killed was statistically analyzed. The mean and the variance of the serving time were calculated and are shown in Table 3. Besides them is shown in the same table the value of the reciprocal of the mean service time for each sample, for it is equal to the average number of whales dissected in an hour, which is called the mean cervice rate μ. Therefore, in the following discussion the numericrical value of $1/T$ will be used as the substitute of μ.

Table 3. Service time.

Sample	Mean T	Variance σ^2	1/T
A	0.35	0.0146	2.90
B	0.39	0.0144	2.60
C	0.34	0.0144	2.94

Unit of time : 1 hr.

Before going into the study about the distribution of th service time, some factors were examined whether they have influence upon the service time. Among the factors that might affect the service time, sex and body length were the only ones the author could avail himself of. On examination body length proved to be an influencing factor, but sex did not. To illustrate this conclusion for sex a contigency table for sample C is given (see Table 4). The value of χ^2 is 1.39 for

Table 4. Difference of service time between sexes.

Service time		1~3	4	5 and over
Frequency	Female	31	21	14
	Male	40	34	13

Unit of time : 5 min.

the table. As the degree of freedom is 2 in this case, the sex difference in the classification is clearly insignificant. As for the effect of body length, Pitman's test was applied. For this test each sample was divided into two subsamples, the longer and the shorter. The averages and the variances of the sub-samples were computed. The values of these statistics for sample C are as follows.

sub-sample	mean	variance	size of sub-sample
the shorter	3.72	0.97	72
the longer	3.95	2.26	84

The value of Pitman's W is 0.092 for this sample. Since

$$\text{Prob}\,(W > 0.092) < 0.01,$$

the difference in service time between the sub-samples is clearly significant. Though body length is found to be an influencing factor upon service time, the difference of the mean service time between the sub-samples is very small compared to the mean of the service time. This is true with the other two samples. One may, therefore, well neglect the effect of body length. Hence there are no factors virtually that affect service time as far as the material is concerned.

One of the common types of service time distribution is Erlangian. If the service time is Erlangian distributed, the probability b(t) that the service time of a unit is t is given by

$$b(t)=(k\mu)^k t^{k-1} e^{-k\mu t}/\Gamma(k),$$

where k is the order of the distribution. Then the mean of the distribution is $1/\mu$ and the variance $1/k\mu$. If k is equal to unity, the distribution is identical with exponential distribution. The Erlang distributions of from the first to the third order were tested whether they would give a good fit to the observed frequency of the service time. All of them, however, fell short of expectation.

4. Waiting time

Waiting time, that is, the time from harpooning till landing showed a wide-ranged distribution in every case. The range covered from about one hour to 23 hours with sample A from three quaters of an hour to 29 hours with sample B and from half an hour to 23 hours with sample C. In table 5 are shown the mean W_0 and the variance of waiting time along with the expected waiting time, which will be explained below.

According to Khintchine, for random input and a given distribution of service time with variance σ^2 and average T, the equation for expected waiting time w at a single service station becomes

$$w = \frac{1}{\lambda}\left[\rho + \frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)}\right] - T,$$

where $\rho = T/\tau = \lambda/\mu$.

By the use of this formula the w's for the samples were evaluated. In the evaluation the variance and the mean of each sample are used as those of its respective population. One finds in Table 5 surprisingly big gaps between the mean waiting time observed and the expected one. The greater part of the gap may be attributable to the service preceding dissection, amongst others to towing. But this point needs further study.

Table 5. Waiting time.

Sample	Mean W_0	Variance	Expected waiting time W	ρ
A	9.42	18.38	1.20	0.85
B	10.02	24.07	0.61	0.75
C	8.35	12.64	0.86	0.82

Unit of time : 1 hr.

5. Conclusion

The present analysis of the whaling data reveals:

1. The occurrence of the whale catch is random, that is, exponential in time distribution.
2. The distribution of service time was studied only of a single stage. It depends on body length but not on sex. An attempt was made to fit the

frequency distribution observed with Erlang distributions. But their fits were too poor to be acceptable.

3. A comparison was made between the waiting time and the corresponding one expected from Khintchine's formula. It showed considerable discrepancy, the greater part of which, the author thinks, may be attributable to the service preceding dissection.